A CONTRIBUTION TO THE THEORY OF STREAMER BREAKDOWN

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The development of a systematic theory of streamer breakdown of a gas requires the consideration of the transport of the region of ionization toward the ionized gas in an electric field depending on the form of the streamer, which in turn is determined by the transport mechanisms [1-3]. In this form the problem is very complicated, and the theory takes the path of investigation of different qualitative models of a streamer [4]. It is assumed in [4] that the rates of anode-directed and cathode-directed streamers are determined by the drift velocity of the electrons. The mechanism of propagation of anode-directed streamers is taken to be the development of avalanche from the leading front of the electrons traveling to the anode. On the side of the cathode, electrons before the front of the cathode-directed streamer are produced due to the transport of radiation from the ionized region [1]. It is shown in [5] that direct photo-ionization is ineffective because of the small range of the quantas, and a mechanism of development of cathode-directed streamer related to the associative ionization of excited atoms is proposed. These atoms are formed by long-span resonance photons from the wings of the spectral line. An interesting prediction of the theory [4] was a linear dependence of the velocity of the streamers on their length. This dependence was confirmed in experiments on the study of streamer breakdown initiated at the center of the discharge gap in spark chambers [6, 7]. At the same time, for streamers developing from avalanche initiated at one of the electrodes the velocity of propagation of the "breakdown wave" remains constant with a good accuracy in gaps having lengths of the order of 1 m. In the present work a qualitative theory is developed which permits one to calculate the velocity of the anode-directed streamer in the case where it is independent of the length. Since for pressures of the order of atmospheric pressure the diffusion coefficient of excited atoms [8] is comparable with the electron diffusion coefficient, the effect of radiation transport is disregarded. The stability of the front of the streamer to infinitely small perturbations is investigated. It is shown that, when the finite thickness of the front is taken into consideration, the streamer is stable. It is unstable in the approximation of infinitely thin leading fronts.

1. Basic Model. We consider the homogeneous problem of propagation of an ionization wave in an electric field directed from the anode to the cathode ($E_x = -E$, $E > 0$). For a qualitative description, we shall assume that the electron mobility μ_e, the diffusion coefficient D_e, the recombination coefficient β, and other nonexponentially varying quantities are constants. In this assumption, taking all the quantities in the steady-state regime to be functions of $\xi = x - ut$ (u is the required velocity of propagation), we have the following system of equations for an anode-directed streamer appearing at the cathode:

$$-u\frac{\partial n_e}{\partial \xi} + \mu_e \frac{\partial}{\partial \xi}(En_e) - D_e\frac{\partial^2 n_e}{\partial \xi^2} = \alpha(T_e)\ \mu_e\ Enn_e - \beta\ n_e n_i \quad (1.1)$$

$$-u\frac{\partial n_i}{\partial \xi} = \alpha(T_e)\mu_e Enn_e - \beta n_e n_i \quad (1.2)$$

$$\frac{\partial E}{\partial \xi} = -4\pi e(n_i - n_e) \quad (1.3)$$

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$$-u\frac{\partial}{\partial\xi}\left(\frac{3}{2}T_e n_e\right)-\frac{\partial}{\partial\xi}\varkappa_e\frac{\partial T_e}{\partial\xi}+\mu_e\frac{\partial}{\partial\xi}\left(En_e\frac{5}{2}T_e\right)=n_e e\mu_e E^2-I\left(\alpha\mu_e Enn_e-\beta n_e n_i\right)-\dot{\varepsilon}\ . \tag{1.4}$$

Here n_e, n_i are concentrations of electrons and ions, $\alpha(T_e)\mu_e En$ is the ionization constant, n is the density of the gas, T_e is the electron temperature, $\varkappa_e$ is the electronic thermal conductivity proportional to n_e, I is the ionization potential, and $\dot{\varepsilon}$ characterizes the energy losses of electrons during collisions with the gas. If the main mechanism is that of elastic losses, then

$$\dot{\varepsilon}\sim\frac{2m_e}{M}\frac{n_e T_e}{\tau_y}$$

where τ_y is the mean free time between elastic collisions; in the case of inelastic losses, $\dot{\varepsilon}\sim\Delta\varepsilon/\tau_{in}$, where $\Delta\varepsilon$ is of the order of the characteristic energy transferred in inelastic collision with frequency τ_{in}^{-1}.

Equations (1.1) and (1.2) describe the balance of the number of electrons and ions (for ions we neglect their mobility and diffusion along the field); (1.3) is Poisson's equation for the electric field ($e > 0$). Equation (1.4) describes the energy balance of the electron gas taking into consideration the energy transport by thermal conductivity as well as in the drift motion of the electrons to the anode. The right-hand side of (1.4) contains joule heating, ionization energy, and energy losses in collisions of electrons with atoms of the gas. In the absence of a field system (1.1)-(1.4) describes the slow ionization wave discussed in [9].

Mathematically the problem of slow ionization wave is akin to the problem of propagation of slow combustion [10]. A rigorous mathematical theory for problems of this type was developed for the first time in [11].

A system of equations analogous to (1.1)-(1.4) was investigated in [12, 13] for the ionization wave in a streamer breakdown. In these works the problem was solved under the assumption that the temperature is constant within the transition layer. This may lead to significant errors, since the ionization constant is an exponential function of the temperature. In the equation of energy balance, terms describing thermal conductivity and energy losses of electrons in collisions with atoms were omitted.

We note that in fields of the order of 10^5 V/cm and at pressures of the order of the atmospheric pressure, the temperature of electrons in the case where the mechanism of elastic losses is predominant is $\gtrsim 10^2$ eV, i.e., significantly greater than the ionization energy. Therefore, inelastic collisions play the main role in the energy balance of electrons. In this case it can be assumed that within the width of the transition region, where effective ionization occurs, the distribution function of electrons is adjusted to the local value of the electric field, and the ionization coefficient $\alpha(T_e)$ is a function of the electric field intensity $\alpha(E)$ at the given point [14]. After this the system of equations (1.1)-(1.3) is separated from Eq. (1.4), and it is sufficient to investigate it for the determination of u and the structure of the transition layer.

In the case of absence of ionization and recombination processes, system (1.1)-(1.3) describes the so-called electric field wave in semiconductors with N-shaped volt–ampere characteristic (see review in [15]). In the case under investigation the ionization and recombination processes are decisive. We shall make use of the following model for the simplification of the problem. Since the streamer propagates in the form of a narrow filament which gets smeared out as a result of relatively slow process of ambipolar diffusion, in a rough approximation the transverse dimension of the streamer can be replaced by some average value r. The basic mechanism of loss of charged particles in the main channel may be assumed to be the diffusion drift to the sides, i.e., instead of the term $\beta n_e n_i$ describing recombination in the right-hand side of Eqs. (1.1),(1.2), we can write n_e/τ, where $\tau\sim r^2/D_a$. This replacement retains the main characteristics of the investigated phenomenon while considerably simplifying the mathematical discussion.

We subtract Eq. (1.1) from (1.2) and make use of (1.3):

$$\frac{u}{4\pi e}\frac{\partial^2 E}{\partial\xi^2}-\mu_e\frac{\partial}{\partial\xi}(En_e)+D_e\frac{\partial^2 n_e}{\partial\xi^2}=0\ . \tag{1.5}$$

Equation (1.5) has an integral. Since for $\xi\to+\infty$, $n_e\to 0$, $E\to E_\infty$, we obtain

$$\frac{u}{4\pi e}\frac{\partial E}{\partial\xi}=\mu_e En_e-D_e\frac{\partial n_e}{\partial\xi}\ . \tag{1.6}$$

Equation (1.6) expresses the law of conservation of the total current made up of the displacement current $\sim u\partial E/\partial\xi$, the conduction current $\sim\mu_e En_e$, and the diffusion current. We shall seek the solution with boundary conditions at $-\infty$:

$$E \to E_0 = \text{const}, \; n_e \to n_{-\infty} \; .$$

It is easily seen from (1.6) that in this case, $n_{-\infty} = 0$. If the energy losses of electrons in the excitation of gas atoms are neglected as in [12, 13], then we obtain $n_{-\infty} \neq 0$. Equation (1.6) admits of such a form of the boundary conditions:

$$E_0 = 0, \; n_{-\infty} \neq 0 \; .$$

However, in the present case this implies $\alpha = 0$, and therefore this approach is inapplicable.

Let us first consider the case where the diffusion term in (1.6) is small compared to the conduction current. [In Eq. (1.1) the diffusion term can be of the order of the difference of two "large" terms $u\partial n_e/\partial\xi$ and $\mu_e \partial/\partial\xi(En_e)$, and it should be retained.] Substituting

$$n_e = \frac{u}{4\pi e \mu_e E} \frac{\partial E}{\partial \xi}$$

from (1.6) into (1.1), we obtain

$$-\frac{u}{\mu_e}\frac{\partial}{\partial\xi}\left(\frac{1}{E}\frac{\partial E}{\partial\xi}\right) + \frac{\partial^2 E}{\partial\xi^2} - \frac{D_e}{\mu_e}\frac{\partial^2}{\partial\xi^2}\left(\frac{1}{E}\frac{\partial E}{\partial\xi}\right) = \alpha(E)\, n \frac{\partial E}{\partial\xi} - \frac{1}{\tau\mu_e E}\frac{\partial E}{\partial\xi} \; . \tag{1.7}$$

Equation (1.7) also has an integral which we write taking account of the conditions at $-\infty$:

$$E(-\infty) = E_0$$

$$\left(1 - \frac{u}{\mu_e E}\right)\frac{\partial E}{\partial\xi} - \frac{D_e}{\mu_e}\frac{\partial}{\partial\xi}\left(\frac{1}{E}\frac{\partial E}{\partial\xi}\right) = \int_{E_0}^{E}\left(n\alpha - \frac{1}{\tau\mu_e E}\right) dE \; . \tag{1.8}$$

Let us consider the condition at $+\infty$:

$$E \to E_\infty = \text{const} \; .$$

It follows from (1.8) that

$$\int_{E_0}^{E_\infty}\left(n\alpha - \frac{1}{\tau\mu_e E}\right) dE = 0 \; . \tag{1.9}$$

Equation (1.9) connects the values of the fields E_0 and E_∞ and has the form of the rule of equal areas. It can be used for the estimate of the breakdown intensity E_* of a gap of length d. Assuming for the sake of definiteness that $\alpha(E)$ has the form $\sim\exp(-A/E)$ [14], we approximately have ($E_\infty = E_*$)

$$\frac{nE_*^2}{A}\alpha(E_*) = \frac{1}{\tau\mu_e}\ln\frac{E_*}{E_0} \; . \tag{1.10}$$

Let us estimate the value of E_0 assuming that the current in the ionized region is $\sim\sigma_0 E_0 r^2$, where σ_0 is the conductivity, and that outside this region it is determined by the displacement current $\sim U dC/dt$, where U is the voltage in the gap and C is the capacitance of the electrode–streamer system

$$C \sim S/4\pi d, \quad dC/dt \sim Su/4\pi d^2, \quad u \sim \mu_e E_* \; .$$

Since E_0 occurs in (1.10) under the logarithm sign, this estimate is completely satisfactory. As a result we have

$$E_0 \sim \frac{U}{d}\frac{S}{r^2}\frac{\mu_e E_*}{4\pi\sigma_0 d}$$
$$n\alpha(E_*)\, d \sim \frac{Ad}{\tau\mu_e E_*^2}\ln\frac{4\pi\sigma_0 d^2 r^2}{\mu_e u S} \; . \tag{1.11}$$

Condition (1.11) is an analog of the Meek [2] and Reter [3] conditions, which in the notation used here have the form

$$n\alpha(E_*)\, d \sim 20 \; .$$

For a practical utilization of (1.11), for τ we can take $\tau \sim 10^{-8}$–10^{-10} sec. The right-hand side of (1.11) can have an order of magnitude without violating the condition

$$n\alpha(E_*)\,d \sim 20 \ .$$

At the same time the right-hand side of (1.11) literally differs from this condition and is amenable to experimental verification.

Condition (1.9) can be generalized to the case where the removal of the particles from the main channel is of a recombination nature. Equation (1.8) admits of a lowering of the order. Using the notation

$$\frac{1}{E}\frac{dE}{d\xi} = y(E)$$

and passing on to dimensionless quantities

$$E = E_\infty \varepsilon, \quad \frac{u}{\mu_e E_\infty} = \varkappa, \quad \xi = \frac{\chi}{n\alpha_0}, \quad y = \eta(\varepsilon)\, n\alpha_0$$

such that

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{d\chi} = \eta(\varepsilon) \ ,$$

we obtain

$$\frac{d\eta}{d\varepsilon} = \gamma\left\{\left(1 - \frac{\varkappa}{\varepsilon}\right) - \frac{1}{\varepsilon\eta}\int\limits_{\varepsilon_0}^{\varepsilon}\left[\sigma(\varepsilon') - \frac{\delta}{\varepsilon'}\right]d\varepsilon'\right\} . \qquad (1.12)$$

Here we have introduced the notation

$$\gamma = \frac{\mu_e E_\infty}{n\alpha_0 D_e}, \quad \alpha(E) = \alpha_0\sigma(E), \quad \delta = \frac{1}{\tau\mu_e E_\infty n\alpha_0} \ .$$

[The condition for neglecting the diffusion term in (1.6) has the form $\gamma \gg 1$.]

In the new variables condition (1.9) becomes

$$\int\limits_{\varepsilon_0}^{1} d\varepsilon\left[\sigma(\varepsilon) - \frac{\delta}{\varepsilon}\right] = 0 \ .$$

The boundary conditions for Eq. (1.12) are: for $\varepsilon = \varepsilon_0$, $\eta = 0$, and for $\varepsilon = 1$, $\eta = 0$, i.e., the integral curve of Eq. (1.12) must pass through the two singular points of this equation. Multiplying (1.12) by $\eta(\varepsilon)$ and integrating over ε from ε_0 to 1 with the boundary conditions taken into consideration, we obtain

$$\int\limits_{\varepsilon_0}^{1} d\varepsilon\, \eta(\varepsilon)\left[\frac{\varkappa}{\varepsilon} - 1\right] = \int\limits_{\varepsilon_0}^{1}\frac{d\varepsilon}{\varepsilon}\int\limits_{\varepsilon_0}^{\varepsilon}\left[\frac{\delta}{\varepsilon'} - \sigma(\varepsilon')\right]d\varepsilon' \ . \qquad (1.13)$$

Condition (1.13) can be used to determine the dimensionless velocity $\varkappa$. We rewrite Eq. (1.12) in the following form:

$$\frac{d\eta}{d\varepsilon} = \gamma\left[\frac{\theta(\varepsilon)}{\varepsilon\eta} - \left(\frac{\varkappa}{\varepsilon} - 1\right)\right], \quad \theta(\varepsilon) = \int\limits_{\varepsilon_0}^{\varepsilon}\left[\frac{\delta}{\varepsilon} - \sigma(\varepsilon)\right]d\varepsilon \ . \qquad (1.14)$$

For ε close to ε_0

$$\theta(\varepsilon_0) = 0, \ \theta(\varepsilon) \approx \theta_0'(\varepsilon - \varepsilon_0), \ \theta_0' \sim \frac{\delta}{\varepsilon_0} \ ,$$

and for ε close to 1

$$\theta(1) = 0, \ \theta(\varepsilon) \approx |\theta_1'|(1 - \varepsilon), \ |\theta_1| \sim \sigma(1) \ .$$

Let us investigate Eq. (1.14) near the singular points $\varepsilon = \varepsilon_0$, $\eta = 0$. We shall seek the solution in the form $\eta = A(\varepsilon - \varepsilon_0)$. Assuming that $\varkappa/\varepsilon_0 \gg 1$, we obtain

$$A = \sqrt{\left(\frac{\gamma\varkappa}{2\varepsilon_0}\right)^2 + \frac{\gamma\theta_0'}{\varepsilon_0}} - \frac{\gamma\varkappa}{2\varepsilon_0} \ . \qquad (1.15)$$

The roots of the characteristic equation have different signs, i.e., the singular point $\varepsilon = \varepsilon_0$, $\eta = 0$ is a saddle point and the desired solution corresponds to the root (1.15). In x space the solution has the form

$$\varepsilon = \varepsilon_0 + C \exp (A\varepsilon_0 x) \ ,$$

and the characteristic thickness of the rear front is $\sim u\tau$.

Near the point $\varepsilon = 1$, $\eta = 0$, Eq. (1.14) becomes

$$\frac{d\eta}{d\varepsilon} = \gamma [\, |\theta_1'| (1 - \varepsilon) - \eta (\varkappa - 1)] \, \eta^{-1} \, . \tag{1.16}$$

The characteristic equation has real roots [if $\gamma(\varkappa - 1)/2 > (\gamma|\theta_1'|)^{1/2}$] of the same sign (nodes):

$$\lambda_{1,2} = -\frac{\gamma(\varkappa-1)}{2} \pm \left\{\left[\frac{\gamma(\varkappa-1)}{2}\right]^2 - \gamma|\theta_1'|\right\}^{1/2} \, . \tag{1.17}$$

Hence we obtain the condition for the velocity

$$\varkappa \geqslant 1 + 2\sqrt{\frac{|\theta_1'|}{\gamma}} \tag{1.18}$$

or in dimensionless form

$$u \geqslant \mu_e E_\infty + 2\sqrt{D_e \mu_e E_\infty n\alpha(E_\infty)} \ .$$

As shown in [11], the velocity

$$u = \mu_e E_\infty + 2\sqrt{D_e \mu_e E_\infty n\alpha(E_\infty)} \tag{1.19}$$

is the limiting velocity for $t \to \infty$ for all monotonic solutions of an equation of this type.

The obtained equation has a simple physical meaning: in the system of coordinates moving with the drift velocity $\mu_e E_\infty$, the ionization wave propagates due to electron diffusion on characteristic scale $\sim (D_e \tau_i)^{1/2}$, where

$$\tau_i \sim [\mu_e E_\infty \, n\alpha \, (E_\infty)]^{-1}$$

is the mean time between ionizing collisions, so that the thickness of the leading front is of the order of $(D_e/n\alpha(E_\infty)\mu_e E_\infty)^{1/2}$, and the characteristic velocity is $\sim (D_e/\tau_i)^{1/2}$, which is reflected in the second term in (1.19). For $\gamma \gg 1$ we have $\mu_e E_\infty \gg (D_e/\tau_i)^{1/2}$. This condition denotes that the velocity of the anode-directed streamer is of the same order of magnitude as the drift velocity.

The diffusion correction to the velocity of the streamer (1.19) cannot exceed the term corresponding to the drift velocity $\mu_e E_\infty$. Considering Eq. (1.4) for $\xi \to \infty$ we obtain an upper estimate for the electron temperature T_e at $+\infty$:

$$eE_\infty / I > \alpha(T_e) n \ . \tag{1.20}$$

Condition (1.20) physically means that only a part of the joule heat liberated before the front is used in ionization. Estimating the diffusion term in equation (1.19) with the help of this inequality and also making use of the relation between the diffusion coefficient and the mobility, we have

$$2\sqrt{D_e \mu_e E_\infty n\alpha(E_\infty)} < 2\mu_e E_\infty \sqrt{T_{e\infty}/I} \, .$$

It is evident from here that the second term in equation (1.19) is always small compared to the first in the conditions of applicability of the present discussion. For this reason the mechanism of electron diffusion cannot ensure propagation of the cathode-directed streamer, and for its investigation it is necessary to consider the transport of radiation.

2. Stability of Streamer Front. An approximate method of solution of the system of equations (1.1)-(1.3) permits one to find the unperturbed state in the problem of stability of the front of the streamer. Here the assumption $\gamma \gg 1$ is not required, and it is easy to apply the method of successive approximations refining the obtained solution. For actual determination of the functions $n_e(\xi)$, $n_i(\xi)$, and $E(\xi)$ with the required accuracy, several iterations must be carried out.

Replacing the field E in Eqs. (1.1) and (1.2) by its asymptotic value at $\pm\infty$, we obtain

$$-u\frac{\partial n_e}{\partial \xi}+\mu_e E(\pm\infty)\frac{\partial n_e}{\partial \xi}-D_e\frac{\partial^2 n_e}{\partial \xi^2}=\alpha[E(\pm\infty)]\mu_e E(\pm\infty)nn_e-\frac{n_e}{\tau} \tag{2.1}$$

$$-u\frac{\partial n_i}{\partial \xi}=\alpha[E(\pm\infty)]\mu_e E(\pm\infty)nn_e-\frac{n_e}{\tau}\ . \tag{2.2}$$

In place of field E we introduce the potential

$$E_x=-\frac{\partial\varphi}{\partial\xi}=-E$$

and write Poisson's equation

$$\frac{\partial^2\varphi}{\partial\xi^2}=-4\pi e(n_i-n_e)\ . \tag{2.3}$$

Equations (2.1)-(2.3) are easily solved for $\xi > 0$ and $\xi < 0$. The solutions thus obtained must be matched at $\xi = 0$, taking into consideration the conditions

$$\begin{gathered} n_e|_{0-}=n_e|_{0+}\\ \left[D_e\frac{\partial n_e}{\partial\xi}+(u-\mu_e E_0)n_e\right]\Big|_{0-}=\left[D_e\frac{\partial n_e}{\partial\xi}+(u-\mu_e E_\infty)n_e\right]\Big|_{0+}\\ n_i|_{0-}=n_i|_{0+},\quad \varphi|_{0-}=\varphi|_{0+},\quad \frac{\partial\varphi}{\partial\xi}\Big|_{0-}=\frac{\partial\varphi}{\partial\xi}\Big|_{0+}\ . \end{gathered} \tag{2.4}$$

The second condition in (2.4) is easily obtained by integrating (2.1) near $\xi = 0$. The relation between the fields at $+\infty$ and $-\infty$ is obtained from Eq. (1.1),

$$\int_{-\infty}^{+\infty}d\xi\left[\alpha(E)\mu_e Enn_e-\frac{n_e}{\tau}\right]=0\ .$$

Finding the solution and substituting it into (2.4), we obtain the condition of solvability of these equations

$$\left[(L_3^{-1}+L_1^{-1})+\frac{\mu_e(E_\infty-E_0)}{D_e}\right]\left(\frac{L_2}{l_2}-\frac{L_3}{l_3}\right)=\left[(L_3^{-1}+L_2^{-1})+\frac{\mu_e(E_\infty-E_0)}{D_e}\right]\left(\frac{L_1}{l_1}-\frac{L_3}{l_3}\right) \tag{2.5}$$

where

$$\begin{gathered} L_{1,2}^{-1}=+\frac{u-\mu_e E_\infty}{2D_e}\pm\left[\left(\frac{u-\mu_e E_\infty}{2D_e}\right)^2-\frac{\alpha(E_\infty)\mu_e E_\infty n-1/\tau}{D_e}\right]^{1/2}\\ L_3^{-1}=-\frac{u-\mu_e E_0}{2D_e}\pm\left[\left(\frac{u-\mu_e E_0}{2D_e}\right)^2+\frac{1/\tau-\alpha(E_0)\mu_e nE_0}{D_e}\right]^{1/2}\\ l_1^{-1}=\left[\alpha(E_\infty)\mu_e nE_\infty-\frac{1}{\tau}\right]u^{-1},\ l_2=l_1,\ l_3^{-1}=\left[\frac{1}{\tau}-\alpha(E_0)\mu_e nE_0\right]u^{-1}\ . \end{gathered}$$

It is assumed that

$$\alpha(E_\infty)\mu_e nE_\infty\gg 1/\tau,\ 1/\tau\gg\alpha(E_0)\mu_e nE_0\ .$$

For

$$u=\mu_e E_\infty+2\sqrt{D_e n\mu_e E_\infty\alpha(E_\infty)}$$

Eq. (2.5) is satisfied identically, since in this case $L_1 = L_2$. This confirms the assumption that the equation obtained for the velocity of the streamer is valid without the use of the condition $\gamma \gg 1$.

Let us now consider the problem of stability of the streamer front. Let the perturbed solution depend on $\xi = x - ut$, t, and y according to the law $\sim\exp(-i\omega t + iky)f(\xi)$. The stability to one-dimensional perturbations that do not depend on y is determined by the method used in [16] in the problem of stability of the front of a flame. Denoting the perturbed quantities by a prime, we obtain a system generalizing systems (2.1)-(2.3):

$$D_e \frac{\partial^2 n_e'}{\partial \xi^2} + \frac{\partial n_e'}{\partial \xi}(u - \mu_e E(\pm\infty)) + n_e'[i\omega - D_e k^2 + \alpha[E(\pm\infty)]\mu_e n E(\pm\infty) - 1/\tau] = 0 \quad (2.6)$$

$$u \frac{\partial n_i'}{\partial \xi} + i\omega n_i' + n_e'\left[\alpha[E(\pm\infty)]\mu_e n E(\pm\infty) - \frac{1}{\tau}\right] = 0 \quad (2.7)$$

$$\partial^2\varphi'/\partial\xi^2 - k^2\varphi' = -4\pi e(n_i' - n_e') \quad (2.8)$$

The perturbations are assumed to die off at $\pm\infty$,and the asymptotic values of the field $E(\pm\infty)$ remain as before. In the unperturbed problem the solutions for $\xi > 0$ and $\xi < 0$ were matched at the front $\xi = 0$. Now the matching should be done at the perturbed boundary

$$\xi' = A' \exp(-i\omega t + iky) \quad (2.9)$$

Solving system (2.6)-(2.8) and matching the solutions at the perturbed boundary (2.9), we obtain the condition of existence of a nontrivial solution

$$\left[\frac{1}{\lambda_+} + \frac{1}{\lambda_-} + \frac{\mu_e(E_+ - E_-)}{D_e}\right]\left[\left(\frac{1}{L_1} + \frac{1}{L_3}\right)\frac{\lambda_-}{l_3(1 + i\omega\lambda_-/u)} - \frac{1}{l_1} - \frac{1}{l_3}\right] =$$
$$= \left[\frac{\lambda_-}{l_3(1 + i\omega\lambda_-/u)} - \frac{\lambda_+}{l_1(1 - i\omega\lambda_+/u)}\right]\left[\frac{1}{L_1}\left(\frac{1}{\lambda_-} + \frac{1}{L_1}\right) + \frac{1}{L_3}\left(\frac{1}{\lambda_-} - \frac{1}{L_3}\right) + \frac{\mu_e(E_+ + E_-)}{DL_1}\right] \quad (2.10)$$

where

$$\lambda_+^{-1} = \left[\frac{\mu_e E_\infty n\alpha(E_\infty)}{D_e}\right]^{1/2} + \left[\frac{k^2}{2} - \frac{i\omega}{2D_e}\right]^{1/2}$$
$$\lambda_-^{-1} = -\frac{u - \mu_e E_0}{2D_e} + \left[\left(\frac{u - \mu_e E_0}{2D_e}\right)^2 + \frac{1/\tau - \alpha(E_0)\mu_e n E_0 + D_e k^2 - i\omega}{2D_e}\right]^{1/2}.$$

For $k = 0$, $\omega = 0$, $\lambda_+ = L_1 = L_2$, $\lambda_- = L_3$ condition (2.10) goes over into (2.5); (2.10) plays the role of a dispersion equation, with which $\omega(k)$ is found and the question of stability is resolved. Let us consider (2.10) in the longwave limit $kL \ll 1$. After some simple computations, we obtain

$$\omega \approx -iD_e k^2 + O(k^4)\ . \quad (2.11)$$

Thus, in the case under investigation the front is stable to infinitely small perturbations. (In the case $kL \gg 1$ the solution is also stable.) Physically, equation (2.11) means that the deformations of the front are reducible due to the diffusion of the particles from the ionized zone.

Let us consider the question of stability of a streamer in the approximation of infinitely thin leading front. Let the velocity of the boundary u(E) satisfy the condition

$$(du/dE)_0 = u'(E_\infty) > 0\ .$$

(In the present case this condition is satisfied.) Outside the boundary ($\xi > 0$) the unperturbed potential has the form: $\varphi = -E_\infty \xi$. We deform the boundary in such a way that the equation of the perturbed boundary has the form (2.9). The perturbations of the potential are determined from Poisson's equation

$$\partial^2\varphi'/\partial\xi^2 - k^2\varphi' = 0$$

from which we obtain

$$\varphi' = B\exp(-k\xi + iky - i\omega t)\ .$$

Requiring that the potential ($\varphi + \varphi'$) be equal to zero at the perturbed boundary, we obtain

$$E_\infty A' = B'\ .$$

The perturbation of the electric field (of the component normal to the boundary) is

$$E' = kB'\exp(-k\xi + iky - i\omega t) = kA'E_\infty \exp(-k\xi + iky - i\omega t)\ .$$

The perturbation of the velocity is given by the equation

$$u' = \frac{d\xi'}{dt} = -i\omega A'\exp(-i\omega t + iky) = \left(\frac{du}{dE}\right)_0 E'.$$

As a result, for the rate of growth of the perturbations, we obtain

$$\omega = ik\left(\frac{du}{dE}\right)_0 E_\infty . \tag{2.12}$$

In contrast to (2.11), equation (2.12) shows that an infinitely thin front is unstable with increment $\gamma \sim$ ku. A similar pattern emerges in the problem of stability of the front of a flame. As shown by Landau [17], a flame considered as a surface of discontinuity is unstable with increment ~ku. At the same time, a consideration of the finite thickness of the front shows [18] that because of the thermal conductivity (neglecting the diffusion of the combustible) the front is stable to infinitely small perturbations. The two investigated approaches do not give solutions smoothly passing from one to the other in the limit, when the wavelength is larger than the width of the front. Apparently, this means that the approximation of infinitely thin front corresponds to the investigation of perturbations whose amplitude is large compared to the thickness of the front (this remark was made by A. A. Vedenov).

As a result it may happen that at the initial stage, when the width of the front is large, the streamer is stable to infinitely small perturbations of the front. At the later stage, when the front becomes thin (this occurs in the case of propagation of a streamer developing far from both electrodes), it is unstable to perturbations larger than the width of the front. Similar physical considerations were developed in [4].

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